Entropy generation of viscous dissipative flow in thermal non-equilibrium porous media with thermal asymmetries

Yi Shen Chee, Tiew Wei Ting, Yew Mun Hung*

School of Engineering, Monash University, 47500 Bandar Sunway, Malaysia

1. Introduction

Forced convective heat transfer in porous media prevails in versatile engineering applications such as electronics cooling, porous insulation, catalytic reactors and flow of liquid in biological and physiological processes. Due to the presence of frictional heating arising from increasing contact of fluid and solid phases and the wall as well as internal heating associated with the mechanical power needed to extrude the fluid through a porous medium, the effect of viscous dissipation is essentially significant as compared to that of clear-fluid flow [1–5]. Viscous dissipation which manifests itself as a source term in the fluid flow induces appreciable rise in fluid temperature due to the conversion of kinetic motion of fluid to thermal energy. Most of the related studies involving viscous dissipation effect in porous-medium flow employed the one-equation model which assumes both the solid and fluid phases to be in locally thermal equilibrium [6–10]. However, the distinctive thermophysical properties of solid phase and fluid phase in a porous medium instigate considerable thermal resistance at the interface between the two phases and induce significant temperature difference between the two phases, invalidating the assumption of local thermal equilibrium [11]. By considering viscous dissipation effect of forced convection in porous medium subjected to uniform wall heat fluxes, the one-equation-model deviates significantly from the two-equation model and the Nusselt number is strongly affected by viscous dissipation [5,12]. On the other hand, studies of asymmetrical thermal boundaries on forced convection heat transfer in porous channel are relatively scarce. By employing the local thermal equilibrium model, Mondal [13] investigated asymmetrical heating and cooling of a porous medium in a parallel plate channel subjected to constant wall temperatures with internal heat generation. Therefore, the issue of coupled effects of thermal asymmetries and local thermal non-equilibrium on forced convection in porous media poses an interesting subject to be addressed and investigated.

Apart from the analysis based on the basic conservation laws, the second-law analysis dealing with entropy generation attributed to thermodynamic irreversibilities is crucial for optimum operating conditions in designing a system with less entropy and exergy destruction. In accordance to the Gouy–Stodola theorem, the loss of the available work of the system is directly proportional to the entropy generation [14]. This type of engineering approach which is known as EGM (Entropy Generation Minimization) is a robust design tool in applied thermal engineering applications [15–19].

* Corresponding author. Tel.: +60 3 5514 6251; fax: +60 3 5514 6207. E-mail address: hung.yew.mun@monash.edu (Y.M. Hung).

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ABSTRACT

The effect of thermal asymmetrical boundaries on entropy generation of viscous dissipative flow of forced convection in thermal non-equilibrium porous media is analytically studied. The two-dimensional temperature, Nusselt number and entropy generation contours are analysed comprehensively to provide insights into the underlying physical significance of the effect on entropy generation. By incorporating the effects of viscous dissipation and thermal non-equilibrium, the first-law and second-law characteristics of porous-medium flow are investigated via various pertinent parameters, i.e. heat flux ratio, effective thermal conductivity ratio, Darcy number, Biot number and averaged fluid velocity. For the case of symmetrical wall heat flux, an optimum condition with a high Nusselt number and a low entropy generation is identified at a Darcy number of $10^{-4}$, providing an ideal operating condition from the second-law aspect. This type of heat and fluid transport in porous media covers a wide range of engineering applications, involving porous insulation, packed-bed catalytic process in nuclear reactors, filtration transpiration cooling, and modelling of transport phenomena of microchannel heat sinks.
Recently, a number of studies dealing with entropy analysis for internal forced convection have been reported [19–27]. However, there exist only limited studies on entropy generation of thermal non-equilibrium porous medium flow. The entropy generation of rarefied gaseous slip flow in micro-porous channel was studied under thermal non-equilibrium condition without considering viscous dissipation in the energy equation [28]. In another numerical modelling, the thermal non-equilibrium model was employed to investigate the entropy generation of natural convection in a saturated porous cavity [29]. On the other hand, the two-energy-equation model with viscous dissipation effect was used and the volume-averaged entropy generation function was developed and analysed numerically [30]. Ting et al. [31] pointed out that the entropy generation is intimately related to the effectiveness of the interstitial heat transfer between the solid and fluid phases of nanofluid flow in porous media, substantiating the significance of thermal non-equilibrium condition in the second-law analysis. This study aims to analyse the forced convection of viscous dissipative flow in porous media subjected to thermal asymmetry from the first-law and the second-law thermodynamic aspects. Particularly, the second-law analysis associated with the effects of thermal asymmetry on forced convection in a thermal non-equilibrium porous media is still unavailable in the existing literature. By incorporating thermal asymmetric boundaries and utilizing thermal non-equilibrium model for forced convection in a porous medium, we obtain the two-dimensional closed-form solutions of temperature profiles of solid and fluid phases and derive the two-dimensional entropy generation analytically by considering the interstitial heat transfer between the solid and fluid phases. With the variations of various pertinent parameters such as Darcy number, Brinkman number and Biot number, the thermal non-equilibrium entropy generation is scrutinized under various cases of asymmetrical heat fluxes. We perform a comprehensive study to delineate the essential attributes of the underlying physical significance of the thermal asymmetries in the entropy generation of viscous dissipative flow in porous media.

### 2. Mathematical formulation

#### 2.1. First-law formulation

The working fluid flows through a porous medium embedded in channel between two parallel plates subjected to asymmetrical thermal boundary conditions as illustrated in Fig. 1. For steady-state fully developed laminar flow in a porous medium between infinitely wide parallel plates, the Brinkman momentum equation is given by

\[
\frac{\mu}{K} \frac{d^2 v_x}{dx^2} + \mu_{eff} \frac{d^2 v_x}{dy^2} - \frac{dP}{dx} = 0, \tag{1}
\]

where \(\mu\) and \(\mu_{eff}\) are the fluid viscosity and the effective viscosity of porous medium, respectively, \(K\) is the permeability, \(v_x\) is the fluid velocity and \(P\) is the pressure. The Forchheimer term is neglected in Eq. (1). This term is only dominant for high Reynolds number flow.

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### Greek symbols

| **\(\varepsilon\)** | porosity of porous medium |
| **\(\mu\)** | fluid viscosity (N s m\(^{-2}\)) |
| **\(\mu_{eff}\)** | effective viscosity (N s m\(^{-2}\)) |
| **\(\rho\)** | fluid density (kg m\(^{-3}\)) |
| **(\(\theta\))** | dimensionless temperature profile |
| **\(\phi\)** | dimensionless bulk mean temperature |

### Subscripts

| **f** | of fluid phase |
| **s** | of solid phase |
| **w1** | of bottom wall |
| **w2** | of top wall |

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where inertial effects are dominant [32]. By introducing the dimensionless variables

\[ Y = \frac{y}{H}, \quad \frac{dP}{dx} = \gamma, \quad V = \frac{\mu v_x}{\gamma H^2}, \quad Da = \frac{K}{H^2}, \quad M = \frac{\mu_{\text{eff}}}{\mu}, \]  

where \( H \) is half of the channel height, \( Da \) is the Darcy number and \( M \) is the viscosity ratio, Eq. (1) can be non-dimensionalized as

\[ M \frac{d^2V}{dY^2} - \frac{1}{Da} V + 1 = 0. \]  

(3)

With the non-slip boundary condition at the walls

\[ Y = \pm 1, \quad V = 0. \]  

(4)

Eq. (3) can be solved to obtain the dimensionless velocity profile as

\[ V = (Da) \left[ 1 - \frac{\cosh(SY)}{\cosh(S)} \right]. \]  

(5)

where \( S \) is the porous medium shape factor given by

\[ S = \frac{1}{\sqrt{MDa}}. \]  

(6)

From Eq. (5), the channel-height averaged dimensionless velocity is given by

\[ \langle V \rangle = (Da) \left[ 1 - \frac{\tanh(S)}{S} \right]. \]  

(7)

while the ratio of fluid velocity profile to average fluid velocity is expressed as

\[ \psi = \frac{v_x}{\langle v_x \rangle} = \frac{V}{\langle V \rangle} = \frac{Scosh(S) - Scosh(SY)}{Scosh(S) - sinh(S)}. \]  

(8)

where \( \langle v_x \rangle \) is the channel-height averaged dimensionalized velocity of the flow. Considering thermal non-equilibrium condition between the two phases and assuming constant thermo-physical properties, the steady-state energy equation of solid and fluid phases are, respectively, given by Refs. [12,33,34],

\[ k_{se} \frac{\partial^2 T_s}{\partial y^2} + h a (T_s - T_f) + \frac{\mu_s^2}{K} + \mu_{\text{eff}} \left( \frac{dV}{dy} \right)^2 = \rho c_p v_x \frac{\partial T_f}{\partial x}. \]  

(10)

where \( k_{se} \) and \( k_{fe} \) are the effective thermal conductivities of solid and fluid phases, respectively, \( h \) is the local interstitial heat transfer coefficient between the two phases, \( a \) is the specific surface area, \( \rho \) is the fluid density and \( c_p \) is the fluid specific heat capacity. The viscous dissipation in porous medium is described by the third and the fourth terms on the left-hand-side of Eq. (10). The thermal boundary conditions at the channel walls are expressed as

\[ q_{w1} = -\left[ k_{se} \frac{\partial T_s}{\partial y} + k_{fe} \frac{\partial T_f}{\partial y} \right]_{y=-H}, \quad q_{w2} = \left[ k_{se} \frac{\partial T_s}{\partial y} + k_{fe} \frac{\partial T_f}{\partial y} \right]_{y=H}. \]  

(11)

The solid, fluid and channel wall are considered to have the same temperature at the interface of the porous medium and channel walls as [11]

\[ T_s|_{y=-H} = T_f|_{y=-H} = T_{w1}. \quad T_s|_{y=H} = T_f|_{y=H} = T_{w2}. \]  

(12)

The summation of Eqs. (9) and (10) yields

\[ \rho c_p v_x \frac{\partial T_f}{\partial x} = k_{se} \frac{\partial^2 T_s}{\partial y^2} + k_{fe} \frac{\partial^2 T_f}{\partial y^2} + \frac{\mu_s^2}{K} + \mu_{\text{eff}} \left( \frac{dV}{dy} \right)^2. \]  

(13)

Since \( \frac{\partial T_f}{\partial x} \) is a constant, integrating Eq. (13) over the channel height gives

\[ \frac{\rho c_p (v_x)}{2H} \frac{\partial T_f}{\partial x} = \int_{-H}^{H} \left[ k_{se} \frac{\partial^2 T_s}{\partial y^2} + k_{fe} \frac{\partial^2 T_f}{\partial y^2} + \frac{\mu_s^2}{K} + \mu_{\text{eff}} \left( \frac{dV}{dy} \right)^2 \right] dy. \]  

(14)

By incorporating the thermal boundary conditions in Eq. (11) into Eq. (14) yields

\[ \frac{\rho c_p (v_x)}{2H} \frac{\partial T_f}{\partial x} = \frac{v}{2H} \left[ q_{w2} + q_{w1} + \int_{-H}^{H} \left( \frac{\mu_s^2}{K} + \mu_{\text{eff}} \left( \frac{dV}{dy} \right)^2 \right) dy \right]. \]  

(15)

The pertinent dimensionless variables are introduced as follows [12]

\[ \theta = \frac{k_{se} [2T - (T_{w1} + T_{w2})]}{H (q_{w1} + q_{w2})}, \quad Bi = \frac{haH^2}{k_{se}}, \quad k_r = k_{fe} \frac{k_{se}}{k_{fe}}. \]  

(16)

\[ Br = \frac{\mu H (v_x)^2}{K (q_{w1} + q_{w2})}, \quad Br = \frac{Br}{S^2} = \frac{\mu_{\text{eff}} (v_x)^2}{H (q_{w1} + q_{w2})}. \]  

By utilizing Eqs. (8), (15) and (16), Eqs. (9) and (10) can be rendered dimensionless as

\[ \frac{d^2 \theta_s}{dy^2} - Bi (\theta_s - \theta_f) = 0, \]  

(17)

\[ k_r \frac{d^2 \theta_f}{dy^2} + Bi (\theta_s - \theta_f) = C_1 \cosh^2(SY) + C_2 \cosh(SY) + C_3. \]  

(18)
Eqs. (17) and (18) are subjected to the following boundary conditions

\[ \theta_s(Y = -1) = \theta_f(Y = -1) = \frac{k_{se}(T_{w2} - T_{w1})}{H(q_{w1} + q_{w2})} \]

\[ \theta_s(Y = 1) = \theta_f(Y = 1) = \frac{k_{se}(T_{w2} - T_{w1})}{H(q_{w1} + q_{w2})}. \]

Eqs. (17) and (18) can be solved analytically to obtain the closed-form temperature distributions of the fluid and solid phases as

\[ \theta_f = D_1 \cosh\left(\frac{B_1}{k_f}(k_f + 1)\right)^{1/2} Y + D_2 \cosh(SY) + D_3 \cosh(2SY) + D_4 Y^2 + D_5 Y + D_6 \]

\[ \theta_s = E_1 \cosh\left(\frac{B_1}{k_f}(k_f + 1)\right)^{1/2} Y + E_2 \cosh(SY) + E_3 \cosh(2SY) + E_4 Y^2 + E_5 Y + E_6 \]

The coefficients in Eqs. (21) and (22) are given by in Appendix A. Under constant heat flux, the wall temperatures vary linearly in the longitudinal direction as

\[ T_{w1} = A_1 X + B_1, \quad T_{w2} = A_2 X + B_2. \]

where

\[ A_i = 2H \frac{\partial T_i}{\partial x}, \quad X = \frac{x}{2H}. \]

With \( D_5 = E_5 \), one of the thermal boundary conditions in Eq. (11) becomes

\[ q_{w2} = \left( k_{se} + k_f \right) \frac{T_{w2} - T_{w1}}{2H} + \left( q_{w1} + q_{w2} \right) \frac{H}{k_{se} + k_f} \times \left[ (E_1 + D_1 k_f) \left( \frac{B_1}{k_f}(k_f + 1) \right)^{1/2} \sinh\left( \left( \frac{B_1}{k_f}(k_f + 1) \right)^{1/2} Y \right) + (E_2 + D_3 k_f) \sinh(S) + (E_3 + D_3 k_f) \sinh(2S) \right] + (2)(E_4 + D_4 k_f) \].

By differentiating Eq. (25) with respect to \( x \), we obtain

\[ A_1 = A_2 = A. \]

and hence Eq. (23) can be expressed as

\[ T_{w1} = AX + B_1, \quad T_{w2} = AX + B_2. \]

By utilizing Eqs. (25)–(27), the difference between the wall temperatures at the inlet is

\[ B_2 - B_1 = (q_{w2}) \left( \frac{2H}{k_{se} + k_f} \right) - (q_{w1} + q_{w2}) \frac{H}{k_{se} + k_f} \times \left[ (E_1 + D_1 k_f) \left( \frac{B_1}{k_f}(k_f + 1) \right)^{1/2} \sinh\left( \left( \frac{B_1}{k_f}(k_f + 1) \right)^{1/2} Y \right) + (E_2 + D_3 k_f) \sinh(S) + (E_3 + D_3 k_f) \sinh(2S) \right] + (2)(E_4 + D_4 k_f) \].

By using Eq. (16), Eq. (27) can expressed as

\[ T = \frac{H(q_{w1} + q_{w2})}{2k_{se}} \theta + B_1 + \frac{B_2}{2} + AX. \]

By differentiating Eq. (29) with respect to \( x \) yields

\[ \frac{\partial T}{\partial x} = \frac{A}{2H}. \]

Therefore, the channel walls and the solid and fluid phases share the same temperature gradient \( A \) along the channel. From Eq. (15), \( A \) can be written as

\[ A = 2H \frac{\partial T}{\partial x} = \frac{q_{w1} + q_{w2}}{\rho c_p(p_x)} \left[ 1 + \frac{2Br \cosh(S)}{\cosh(S) - \sinh(S)} \right]. \]

Following this, the wall temperatures \( T_{w1} \) and \( T_{w2} \) as well as the solid and fluid phases temperature fields \( T_f \) and \( T_s \) can be obtained by specifying the fluid inlet temperature. The effective convective heat transfer coefficient of bottom wall is defined as

\[ h_{eff} = \frac{q_{w1}}{T_{w1} - \left\langle T_f \right\rangle}. \]

where

\[ \left\langle T_f \right\rangle = \frac{[H(q_{w1} + q_{w2})]}{2k_s} \left\langle \theta_f \right\rangle + \frac{(T_{w1} + T_{w2})}{2}. \]

The non-dimensional bulk mean temperature can be expressed as

\[ \left\langle \theta_f \right\rangle = \frac{1}{2(V)} \int_{-1}^{1} \text{V}\theta_f \text{d}Y = D_1 Z_1 + D_2 Z_2 + D_3 Z_3 + D_4 Z_4 + D_6, \]
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The effective Nusselt number can be expressed as

$$\frac{Nu}{k_{fe}} = \frac{(4H)/q_{w1}}{k_{fe}} = \frac{T_{w1} - T_{w2}}{k_{fe}}(q_{w1} + q_{w2})/\theta_f.$$  

(36)

In the case of symmetrical heating where \(q_{w1} = q_{w2}\) and \(T_{w1} = T_{w2}\). \(Nu\) is simplified to \(Nu = -2/(k_{fe}/\theta_f)\). Table 1 lists the Nusselt number correlations for various special cases. For clear fluid flow and slug flow with symmetrical heating, the Nusselt number concurs with those reported in Refs. [2, 12]. It should be noted that for clear fluid flow model, the coefficient of \(Br\) in the denominator is 54, which is 2 times the coefficient of 27 found in literature. This is because \(Br\) and \(Ar\) are scaled with double the wall heat flux in the present study according to Eq. (16). Nusselt numbers of special cases including the clear fluid and slug flow models for single wall heating where the total wall heat flux \(q_{w,total}\) is transported through lower wall 1 with upper wall 2 insulated are also depicted. It is found that for \(Bi \to 0, k_r \to 0, Nu_{SWH} = 2Nu_{SYM}\) where \(Nu_{SWH}\) is the Nusselt number for single wall heating and \(Nu_{SYM}\) is the Nusselt number for symmetrical heating. Therefore, the heat transfer efficiency from a single wall is equivalent to the combined heat transfer efficiency of both lower and upper walls for symmetrical heating as \(Bi \to 0\) and \(k_r \to 0\). This is because as \(Bi \to 0\) and \(k_r \to 0\), more heat is transferred to the insulated wall, causing its temperature to be similar to the single heating wall. The similar wall temperatures in turn result in symmetrical temperature profiles. This is reflected in the modified clear fluid and slug flow models for single wall heating presented in Table 1 when \(k_r \to 0\).

### 2.2. Second-law formulation

The entropy generation for the fluid phase is given by Ref. [14].

$$\frac{\rho}{D_s} = \frac{\nabla \cdot q_{out}}{T_f} + \frac{\rho}{D_s} \frac{D_s}{T_f} - \frac{q_{out}}{T_f} \frac{\nabla T_f}{T_f} + \frac{\rho}{D_s} \frac{D_s}{T_f}.$$  

(37)

where \(q_{out}\) is the net heat loss from a control volume through conduction across the control surfaces. The local inter-phase convective heat transfer is not considered in \(q_{out}\) because it occurs within the control volume without crossing any control surface. In the present study, Eq. (37) can be written as

$$\frac{\rho}{D_s} = \frac{1}{T_f} \left[ - \frac{k_{fe}}{\theta_f^2} + \frac{k_{fe}}{T_f} \left( \frac{\partial T_f}{\partial x} \right)^2 + \frac{\partial T_f}{\partial y} \right] + \frac{\rho}{D_s} \frac{D_s}{T_f}.$$  

(38)

Eq. (42) concurs with the total entropy generation expressions reported by Hooman et al. [35] and Betchen and Straatman [30]. The first and second terms on the right-hand-side of Eq. (42) are the entropy generation due to heat conduction within the fluid and solid phases respectively. The third term is entropy generation induced by the convective heat transfer at the interface of the solid and fluid phases while the fourth term is entropy generation contributed by the fluid friction in a porous medium. Eq. (42) can be expressed in non-dimensional form as

$$\frac{D_s}{T_f} = \frac{\rho}{T_f} \frac{D_s}{T_f} = \frac{1}{T_f} \left[ - k_{fe} \frac{\partial T_f}{\partial x} \right] + \frac{1}{T_f} \left[ \frac{k_{fe}}{T_f} \left( \frac{\partial T_f}{\partial x} \right)^2 + \frac{\partial T_f}{\partial y} \right] + \frac{\rho}{D_s} \frac{D_s}{T_f}.$$  

(39)

$$\frac{D_s}{T_f} = \frac{1}{T_f} \left[ - k_{fe} \frac{\partial T_f}{\partial x} \right] + \frac{1}{T_f} \left[ \frac{k_{fe}}{T_f} \left( \frac{\partial T_f}{\partial x} \right)^2 + \frac{\partial T_f}{\partial y} \right] + \frac{\rho}{D_s} \frac{D_s}{T_f}.$$  

(40)

$$\frac{D_s}{T_f} = \frac{1}{T_f} \left[ - k_{fe} \frac{\partial T_f}{\partial x} \right] + \frac{1}{T_f} \left[ \frac{k_{fe}}{T_f} \left( \frac{\partial T_f}{\partial x} \right)^2 + \frac{\partial T_f}{\partial y} \right] + \frac{\rho}{D_s} \frac{D_s}{T_f}.$$  

(41)

Table 1

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<tr>
<th>Symmetrical heating: (q_{w1} = q_{w2})</th>
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<tbody>
<tr>
<td>1 Clear fluid model (S \to 0, Bi \to 0, k_r \to \infty) Nu = 70(17 + 54Br)</td>
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<td>2 Slug flow model (S \to 0, Bi \to 0, k_r \to \infty) Nu = 6</td>
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<tr>
<td>3 Clear fluid model (S \to 0, Bi \to 0, k_r \to \infty) Nu = 70(26 + 27Br)</td>
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<tr>
<td>4 Modified clear fluid model (S \to 0, Bi \to 0, k_r \to \infty) Nu = 70(8.5 + 27Br)</td>
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<td>5 Slug flow model (S \to \infty, Bi \to 0, k_r \to \infty) Nu = 3</td>
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<tr>
<td>6 Modified slug flow model (S \to \infty, Bi \to 0, k_r \to \infty) Nu = 12</td>
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with the term containing the material derivative of specific entropy expressed as
expressed, respectively, as

$$S_{\text{gen}} = \frac{S_{\text{gen}} H}{k_f} = \left[ 1 + \frac{2Br \cosh(S)}{\sinh(S)} \right]^2 \left[ \frac{St}{\psi_\theta + 1} \right]^2 + \left[ \frac{\psi}{\psi_\theta + 1} \right]^2 \left[ \frac{\partial \theta_Y}{\partial Y} \right]^2 \left[ \frac{1}{k_C} \right] \left[ 1 + \frac{2Br \cosh(S)}{\sinh(S)} \right]^2 \\
\times \left[ \frac{St}{\psi_\theta + 1} \right]^2 + \left[ \frac{\psi}{\psi_\theta + 1} \right]^2 \left[ \frac{\partial \theta_Y}{\partial Y} \right]^2 \\
+ \left( \frac{Bi}{k_C} \right) (\theta_i - \theta_l) \left[ \frac{\psi}{\psi_\theta + 1} \right] + \left[ \frac{2Br}{k_C} \right] \left[ \frac{\psi}{\psi_\theta + 1} \right] \\
\times \left[ \frac{\sinh(S) - \sinh(S)}{\cosh(S) - \sinh(S)} \right] + \left[ \frac{S^2 \sin^2(S)}{\cosh(S) - \sinh(S)} \right]^2 \\
\right].$$

(43)

where

$$St = \frac{q_{w1} + q_{w2}}{(\rho c_p (\nu_C / (T_{w1}(X) + T_{w2}(X)))}, \quad \psi = \frac{H(q_{w1} + q_{w2})}{(k_f / (T_{w1}(X) + T_{w2}(X)))}. \tag{44}$$

By using Simpson’s 3/8 rule, Eq. (43) is averaged over the channel height $Y$ to obtain $S_{\text{gen,Y}}$ as a function of $X$. The resulting expression is averaged over the channel length to obtain $S_{\text{gen}}$, which is the average total entropy generation of the system, contributed by two components, i.e. the HTI (heat transfer irreversibility) and the FFI (fluid friction irreversibility) which can be expressed, respectively, as

$$HTI = \left[ 1 + \frac{2Br \cosh(S)}{\sinh(S)} \right]^2 \left[ \frac{St}{\psi_\theta + 1} \right]^2 + \left[ \frac{\psi}{\psi_\theta + 1} \right]^2 \left[ \frac{\partial \theta_Y}{\partial Y} \right]^2 \left[ \frac{1}{k_C} \right] \left[ 1 + \frac{2Br \cosh(S)}{\sinh(S)} \right]^2 \\
\times \left[ \frac{St}{\psi_\theta + 1} \right]^2 + \left[ \frac{\psi}{\psi_\theta + 1} \right]^2 \left[ \frac{\partial \theta_Y}{\partial Y} \right]^2 + \left( \frac{Bi}{k_C} \right) (\theta_i - \theta_l) \\
\times \left[ \frac{\psi}{\psi_\theta + 1} \right] + \left[ \frac{2Br}{k_C} \right] \left[ \frac{\psi}{\psi_\theta + 1} \right]. \tag{45}$$

$$FFI = \frac{(2Br)(\psi)}{k_C} \left( \frac{\psi}{\psi_\theta + 1} \right) \left[ \frac{\cosh(S) - \cosh(S)}{\cosh(S) - \sinh(S)} \right]^2 + \left[ \frac{S^2 \sin^2(S)}{\cosh(S) - \sinh(S)} \right]^2. \tag{46}$$

The local Bejan number which is the ratio of the local heat transfer irreversibility to the local total entropy generation, can be written as $Be = HTI / S_{\text{gen}}$. By averaging the heat transfer

![Graph](image-url)

**Fig. 2.** The variations of (a) $Nu$, (b) $S_{\text{gen,Y}}$, and (c) $Be_Y$, plotted as a function of Darcy number for $\kappa = 0.01$ and $Bi/k_C = 175$ of symmetrical heat-flux case ($q_r = 1$).
Fig. 3. The two-dimensional contour plots of (a) $S_{gen}(X,Y)$, (b) $Re(X,Y)$, and (c) $T_{f}(X,Y)$ for $BrDa = 10^{-10}$, $h_{t} = 0.01$, and $Bi/k_{t} = 175$ at different Darcy numbers of symmetrical heat-flux case ($q_{r} = 1$).
irreversibility over the entire domain of the system, we obtain \(HTV_X\) which is the average total heat transfer irreversibility of the system and the averaged Bejan number, \(Be_{V_X}\) can be expressed as

\[
Be_{V_X} = \frac{HTV_X}{S_{genX}}
\]  

(47)

Both \(S_{genX}\) and \(Be_{V_X}\) which are evaluated numerically using 2-dimensional Simpson’s 3/8 rule have a typical estimated error of \(O(10^{-3})\) [36].

3. Results and discussion

3.1. Input parameters

Water is used as the working fluid. The input parameters used for the computation are listed in Table 2. The variations of thermophysical properties with temperature are assumed to be negligible. By specifying the minimum fluid inlet temperature as 300 K at \(X = 0\) and \(Y = Y_c\) where \(\partial T/\partial Y = 0\), Eq. (29) can be written as

\[
B_1 = 300 - \frac{H(q_{w1} + q_{w2})}{2k_{se}} \theta_f(Y_c) - \frac{(B_2 - B_1)}{2}.
\]  

(48)

which can be used in conjunction with Eq. (28) to specify \(B_1\) and \(B_2\). For asymmetrical heating, the estimated error of \(Y_c\) which is evaluated by secant method is of \(O(10^{-10})\).

In the literature, several expressions of Reynolds number for the flow through porous media are defined based on the porous medium’s microscopic characteristic length [37] with critical values that vary greatly between different microscopic geometries [38]. The Reynolds number based on the pore diameter is given by \(Re_p = 1/\epsilon(\rho \nu D_p/\mu)\) [38], where \(D_p\) is the pore diameter. It should be noted that high-grade foams with small pore diameters have a “laminarising effect” on fluid flow [38]. This is because the characteristic length scale is based on the very small pore diameter, \(D_p\), instead of the channel. For the sake of simplicity, the Reynolds number which is characterized by the hydraulic diameter of the channel is used in the present study and can be written as

\[
Re = \frac{\rho \nu (2H)}{\mu} = \frac{\rho \nu (4H)}{\mu}.
\]  

(49)

The flow regime is confined in the laminar flow region in the range of \(625 \leq Re \leq 6250\) and \(10^{-11} \leq Br Da \leq 10^{-9}\), where \(Br Da\) is proportional to the square of average fluid velocity, \(<\nu>D_p^2\). For this specific range of Reynolds number, the corresponding maximum \(Re_p\) based on the pore diameter is estimated to be in the order of magnitude of \(10^2\), in which case the flow can be safely characterized as laminar flow. By considering viscous dissipation effect and thermal non-equilibrium condition, the first-law and second-law characteristics of porous-medium flow are investigated by varying several pertinent parameters, i.e. heat flux ratio \(q_w = q_{w1}/q_{w2}\), Darcy number \(Da\), \(Br Da\), effective thermal conductivity ratio \(k_r = k_{se}/k_{se}\) and local interstitial Nusselt number between two phases, \(Nu_0 = Bi/k_c\).

3.2. Case of uniform wall heat flux \((q_w = 1)\)

From Fig. 2(a), it is observed that the variation of \(Nu\) with \(Da\) is insignificant at small \(Da\) values and decreases with increasing \(Da\) at large \(Da\) values above \(10^{-5}\). This is because the transverse fluid temperature gradient becomes significant at \(Da\) values above \(10^{-5}\) where the porous medium is less efficient at transferring heat to the fluid as shown in Fig. 3(c). The fluid temperature profile and \(Nu\) are independent of the fluid velocity or \(Br Da\) which affects only the temperature gradient along the channel. From Fig. 2(b), it is observed that \(S_{genX}\) increases with \(Br Da\) and decreases exponentially with \(Da\) until around \(Da = 10^{-4}\), above which \(S_{genX}\) starts converging for all \(Br Da\) values and increases slightly with an increase in \(Da\). This sudden change in the trend of \(S_{genX}\) occurs because the dominance of heat transfer irreversibility increases rapidly for \(10^{-5} < Da < 10^{-4}\) which is indicated by a drastic increase in \(Be_{V_X}\) for \(10^{-5} < Da < 10^{-4}\) in Fig. 2(c).

For \(Da < 10^{-4}\), entropy generation is dominated by fluid friction irreversibility as shown in Fig. 3(a) and (b). Entropy generation

\begin{figure}
\centering
\includegraphics[width=\textwidth]{Fig_4.png}
\caption{The variations of (a) \(Nu\), (b) \(S_{genX}\) and (c) \(Be_{V_X}\) plotted as a function of interstitial Nusselt number \(Nu_0 = Bi/k_c\), for \(Da = 10^{-3}\) and \(Br Da = 10^{-10}\) of symmetrical heat-flux case \((q_w = 1)\).}
\end{figure}
Fig. 5. The two-dimensional temperature contour plots of fluid phase, $T_f(X,Y)$, of (a) $k_r = 0.001$ and (b) $k_r = 0.05$, for $Da = 10^{-5}$ and $BrDa = 10^{-10}$ at different values of interstitial Nusselt number $Nu_0 = Bi/k_r$ of symmetrical heat-flux case ($q_r = 1$).
increases when fluid velocity or BrDa increases because there is no more fluid friction and decreases when Da increases because there are fewer obstructions in the porous medium and less fluid friction. For Da > 10^{-4}, heat transfer irreversibility becomes dominant as shown in Fig. 3(a) and (b) and increases slightly with increasing Da. The increase in Da corresponds to a decrease in the heat transfer efficiency of the porous medium as indicated by Fig. 2(a), causing the heat transfer irreversibility to increase. The two suggested distinct trends are further supported by Fig. 3(a) and (b) where fluid friction irreversibility is dominant and very high at the walls for Da = 10^{-10} whereas heat transfer irreversibility is dominant with a corresponding Be(X,Y) value of 1 throughout the system for Da = 10^{-4}. It should be noted that S_{gen} is minimum in the vicinity of Da = 10^{-4} in Fig. 2(b).

Transverse thermal conduction and its associated entropy generation is maximum at the walls and zero at Y = 0. On the other hand, interstitial heat transfer is maximum at Y = 0 and zero at the walls. Therefore, it is observed from Fig. 3(a) for Da = 10^{-1} that the maximum entropy generation due to interstitial heat transfer at Y = 0 is slightly higher than the maximum entropy generation due to transverse thermal conduction at the walls. The two peaks at around Y = ±0.6 are due to the combined entropy generation of transverse thermal conduction and interstitial heat transfer.

From Fig. 4(a), Nu increases with increasing Bi/kr and decreasing k_r. At large k_r values, Nu increases with increasing Bi/kr until it reaches an asymptotic value which in turn increases with decreasing k_r. This is because a large k_r value corresponds to a low solid phase thermal conductivity which limits the heat conducted from the walls to the porous medium. Therefore, an increase in Bi/kr which corresponds to an increase in the convective heat transfer from the solid to the fluid causes Nu to reach a low asymptotic value limited by the large k_r. From Fig. 4(b) and (c), S_{gen} and Be_{X} decrease with increasing k_r and Bi/kr until they reach their respective asymptotic values which in turn increase with increasing k_r.

From Fig. 5(a), it is observed that an increase in Bi/kr at k_r = 0.001 greatly reduces the fluid’s transverse temperature gradient because the heat transfer rate from the solid to the fluid increases significantly, resulting in a larger Nu smaller S_{gen} and Be_{X}. From Fig. 5(b), an increase in Bi/kr at k_r = 0.05 results only in a slight decrease in the fluid’s transverse temperature gradient. This is because a large k_r value limits the improvement in heat transfer rate from the solid to the fluid with an increase in Bi/kr, resulting in a small asymptotic value for Nu and large asymptotic values for S_{gen} and Be_{X}. Therefore, the heat transfer rate from the porous medium to the fluid is limited by large k_r values, causing the corresponding asymptotic Nu value to decrease and the asymptotic S_{gen} and Be_{X} values to increase when the transverse temperature gradients of fluid are high even at large Bi/kr. It should be noted that a high heat transfer rate or small k_r and large Bi/kr decreases heat transfer irreversibility and also S_{gen} when the former is significant at large Da values. On the other hand, a low heat transfer rate or large k_r and small Bi/kr decreases S_{gen} at small Da values where heat transfer irreversibility is insignificant and fluid friction is very high at the walls as shown in Fig. 3(a). This trend is attributed to a decrease in fluid friction irreversibility at the walls which is given by the last term in Eq. (42), i.e. \((\mu_{eff})T\left(\frac{dv_x}{dy}\right)^2\), due to higher wall temperatures as shown in Fig. 5(b).

From Fig. 6(a), it is observed that the variation of S_{gen} with Bi/kr is relatively insignificant for Da ≤ 10^{-6}. For Da > 10^{-5} where heat transfer irreversibility is dominant, S_{gen} decreases with increasing Bi/kr due to the decrease in heat transfer irreversibility. It is also observed that the reduction in S_{gen} with increasing Bi/kr is more significant at larger Da values where thermal effects are more dominant. From Fig. 7(a), it is observed that for Bi/kr = 50, the entropy generation is relatively high at around Y = 0 where entropy generation due to interstitial heat transfer is dominant. For Bi/kr = 300, the entropy generation is relatively high at the walls where entropy generation due to transverse thermal conduction is dominant. Therefore, an increase in Bi/kr significantly decreases entropy generation due to interstitial heat transfer and S_{gen} at large Da values. This is because a large Bi/kr decreases the temperature difference between the solid and fluid phases.

From Fig. 6(b), the variation of S_{gen} with Bi/kr is relatively insignificant for BrDa ≥ 5 × 10^{-10}. This is because fluid friction irreversibility which is independent of Bi/kr, is dominant. For BrDa ≤ 10^{-10}, S_{gen} decreases with increasing Bi/kr until it reaches an asymptotic value with the decrease being more significant at smaller BrDa values. This is because for a slower fluid flow, the system’s longitudinal temperature gradient is larger as the fluid receives more heat from the walls and leaves the channel at a higher temperature. Hence, the slower fluid flow increases the overall temperature of the system due to the constant wall heat flux boundary condition and decreases S_{gen}. This is illustrated by the cone-like patterns in Fig. 7(b) becoming elongated at a larger BrDa which indicates a corresponding decrease in variation of S_{gen}(X,Y) along the channel. Therefore, an increase in Bi/kr is expected to decrease S_{gen} significantly at small BrDa values where heat transfer and the longitudinal temperature gradient has a dominant effect on the system’s overall temperature.

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**Fig. 6.** The plots of S_{gen} as a function of interstitial Nusselt number Nu_{0} = Bi/kr with (a) Da (BrDa = 10^{-10}) and (b) BrDa (Da = 10^{-5}), being a parameter for k_r = 0.01 of symmetrical heat-flux case (q_f = 1).
3.3. Case of asymmetrical heating (0 ≤ qr ≤ 1)

From Fig. 8(a), it is observed that $Nu$ increases with increasing $qr$ at different $Da$ values. The relative $Nu$ values at different $Da$ over the range of $qr$ are also consistent with the observations for symmetrical heating with $Nu$ being significantly lower at $Da > 10^{-3}$. It is also observed that at smaller $Da$ values, $Nu$ increases more rapidly with increasing $qr$. In Fig. 9, from the fluid temperature contour plots with an insulated top wall, the transverse temperature gradient in the fluid increases significantly from $Da = 10^{-5}$ to $Da = 10^{-1}$, indicating a corresponding decrease in $Nu$. From Fig. 8(b), the variation of $S_{genYX}$ with $qr$ is relatively insignificant at $Da = 10^{-5}$ where fluid friction irreversibility is dominant. At $Da > 10^{-4}$, $S_{genYX}$ decreases with increasing $qr$ until it reaches an asymptotic value and follows the same trend for all $Da$ values. Therefore, the effect of $qr$ on $S_{genYX}$ is more significant at large $Da$ values where thermal effects are dominant. It should be noted that $S_{genYX}$ is minimum for $qr = 1$, which is the case for symmetrical heating where the temperature gradients are symmetrical and result in less entropy generation. Similar to the case of symmetrical heating, $S_{genYX}$ increases slightly with increasing $Da$ for $Da > 10^{-4}$.

From Fig. 10(a) and (b), it is observed that $S_{genYX}$ and $Be_{YX}$ decrease with increasing $qr$ because symmetrical heating minimises heat transfer irreversibility. It is also observed that the decrease in $S_{genYX}$ and $Be_{YX}$ with increasing $qr$ become less significant at small $k_r$ values. This is because the wall temperatures are almost identical at small $k_r$ values and the temperature profiles have high symmetry even for $qr = 0$, as shown in Fig. 11(a). High solid phase thermal conductivity causes the $S_{genYX}$ and $Be_{YX}$ values to be similar to those for $qr = 1$. At a large $k_r$ value of 0.05, there is a significant difference between the wall temperatures, as shown in Fig. 11(a) due to the low solid phase thermal conductivity. This results in the $S_{genYX}$ and $Be_{YX}$ values to be larger than those for $qr = 1$.

From Fig. 12(a) and (b), it is observed that $S_{genYX}$ and $Be_{YX}$ decrease with increasing $qr$ at various $Bi/k_r$ values in the same manner. Therefore, $Bi/k_r$ does not have an effect on the trend of $S_{genYX}$ and $Be_{YX}$ over a range of $qr$ values. However, $S_{genYX}$ and $Be_{YX}$ decrease slightly with an increase in $Bi/k_r$ because entropy generation due to interstitial heat transfer decreases. From Fig. 11(b), at $Bi/k_r = 300$, the fluid temperature profile is more skewed with a relatively low transverse temperature gradient. This indicates that at a large $Bi/k_r$, the fluid temperature profile resembles the solid temperature profile more closely. Therefore, the small temperature difference between the solid and fluid at large $Bi/k_r$ decreases entropy generation due to interstitial heat transfer for asymmetrical heating.

From Fig. 13(a), when $qr$ increases for $Bi/k_r = 175$, $Nu$ increases at $k_r > 0.01$ and decreases at $k_r < 0.01$. This indicates that $Nu$ decreases
Fig. 8. The variations of (a) $\text{Nu}$, (b) $S_{\text{gen}YX}$ plotted as a function of heat flux ratio $q_r$, for $BrDa = 10^{-10}$, $k_r = 0.01$ and $Bi/k_r = 175$.

Fig. 9. The two-dimensional temperature contour plots of fluid phase, $T_f(X,Y)$, for $BrDa = 10^{-10}$, $k_r = 0.01$ and $Bi/k_r = 175$ at different values of Darcy number of asymmetrical heat-flux case when the upper wall is insulated ($q_r = 0$).

Fig. 10. The variations of (a) $S_{\text{gen}YX}$, and (b) $B_{\text{gen}X}$ plotted as a function of heat flux ratio $q_r$, with $k_r$ being a parameter for $Da = 10^{-5}$, $BrDa = 10^{-10}$, and $Bi/k_r = 175$.
Fig. 11. The two-dimensional temperature contour plots of fluid phase, \( T_f(X,Y) \), of (a) \( Bi/k_t = 175 \) at different \( k_t \) and (b) \( k_t = 0.01 \) at different \( Bi/k_t \) for \( Da = 10^{-5} \) and \( Br Da = 10^{-10} \) of asymmetrical heat-flux case when the upper wall is insulated (\( q_r = 0 \)).
At very small \( k_r \) and \( Bi/k_r \) values, the heat transfer efficiency of the system for \( q_r = 0 \) approaches that of \( q_r = 1 \) \((Nu\text{ for } q_r = 0 \text{ approaches } 2Nu\text{ for } q_r = 1 \text{ as discussed earlier})\) as shown in Fig. 14(a). The corresponding fluid temperature field shown in Fig. 14(b) is similar to that of a system subjected to symmetrical heating, indicating that the single wall’s heat transfer efficiency is similar to the combined heat transfer efficiency of the top and bottom walls for symmetrical heating. This is because a small \( k_r \) results in high heat transfer rate through the solid by thermal conduction whereas a small \( Bi/k_r \) results in low heat transfer rate from the solid to the fluid. As a result, sufficient heat is conducted through the solid to the insulated wall to maintain it at a temperature similar to the single heating wall. The similar wall temperatures result in highly symmetrical temperature profiles, similar to the case of symmetrical heating.

From Fig. 15(a), it is observed that the variation of \( S_{gen}^{XY} \) with \( q_r \) is relatively insignificant at \( BrDa \approx 5 \times 10^{-10} \) where fluid friction irreversibility is dominant as shown in Fig. 15(b). From Fig. 15(a), \( S_{gen}^{XY} \) decreases with increasing \( q_r \) at \( BrDa \leq 10^{-10} \) with the decrease being more significant at smaller \( BrDa \) values. Similar to the case of symmetrical heating, more heat is transferred from the lower wall to the fluid at lower \( BrDa \) values which increases the longitudinal temperature gradient and the system’s overall temperature and decreases \( S_{gen}^{XY} \). From Fig. 16(a) and (b), it is observed that the fluid and solid temperature fields have higher longitudinal temperature gradients at a smaller \( BrDa \) value. Similar effects are observed for the variation of \( S_{gen}^{XY} \) along the channel where the parabolic contours at \( BrDa = 10^{-11} \) become elongated into horizontal strips at \( BrDa = 10^{-9} \) in Fig. 18(c), suggesting that thermal effects are more significant at small \( BrDa \) or low fluid velocities.

4. Conclusions

Force convection in porous media with thermal asymmetries incorporating the viscous dissipation and thermal non-equilibrium effects is analysed analytically from the aspects of the first law and the second law of thermodynamics. The Nusselt number and entropy generation predicted in this study would be a useful analytical tool for a porous-medium system design and performance analysis. The present investigation provides interesting insights into the asymmetrical thermal effects of the solid wall for forced convection in porous media, which are commonly neglected in the existing literature. The Nusselt number decreases with the Darcy number when \( Da > 10^{-3} \), corresponding to the region of the drastic intensification of heat transfer irreversibility. The system operates at optimum condition in the vicinity of \( Da \approx 10^{-4} \) where the Nusselt number is relatively high and the entropy generation is minimized. By decreasing \( k_r \) and increasing \( Bi/k_r \), the thermal performance and the exergetic effectiveness of the flow can be enhanced when heat transfer irreversibility is dominant. On the other hand, increasing \( k_r \) and decreasing \( Bi/k_r \) would improve the second-law performance of the flow when fluid friction irreversibility is dominant. For the case of asymmetrical heating, the Nusselt number decreases with the decrease of \( q_r \) except when \( k_r \) and \( Bi/k_r \) are small. For \( Br = 0 \) and \( k_r \to \infty \), the heat transfer coefficient of the flow with \( q_r = 0 \) is nearly two-fold of that for \( q_r = 1 \), indicating a single wall heating condition that is as effective as symmetrical heating. The effect of asymmetrical heating is significant on the entropy generation when \( BrDa \) is small due to longitudinal temperature variation. This study details a comprehensive investigation that interrelates the two-dimensional temperature, Nusselt
Fig. 13. The plots of $Nu$ as a function of heat flux ratio $\phi_r$ for (a) $Bi/k_r = 175$, $k_r$ being a parameter, (b) $k_r = 0.01$, (c) $k_r = 0.005$ and (d) $k_r = 0.03$, with $Bi/k_r$ being a parameter for $Da = 10^{-5}$ and $BrDa = 10^{-3}$. 
Fig. 14. The plots of (a) Nusselt number $Nu$ as a function of heat flux ratio $q_r$, and (b) two-dimensional temperature contour of fluid phase, $T_f(X,Y)$, of asymmetrical heat-flux case when the upper wall is insulated ($q_u = 0$), for $Da = 10^{-5}$, $BrDa = 10^{-10}$, $k_r = 0.001$ and $Bi/k_r = 50$.

Fig. 15. The variations of (a) $S_{genYX}$ and (b) $BeYX$ plotted as a function of heat flux ratio $q_r$, with $BrDa$ being a parameter for $Da = 10^{-5}$, $Bi/k_r = 175$, and $k_r = 0.01$. 
Fig. 16. The two-dimensional contour plots of (a) fluid-phase temperature $T(X,Y)$, (b) solid-phase temperature and (c) entropy generation $S_{gen}(X,Y)$, at different $BrDa$ for $Da = 10^{-2}$, $k_r = 0.01$ and $Bi/k_r = 175$ of asymmetrical heat-flux case with the upper wall insulated ($q_r = 0$).
number and entropy generation contours to provide insights into the essential attributes of the underlying physical significance of the effects of viscous dissipation, thermal non-equilibrium condition and thermal asymmetrical boundaries on the forced convection in porous media.

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Appendix A

The coefficients in Eqs. (21) and (22) are expressed as

\[ D_2 = \frac{(C_2)(Bi - S^2)}{(S^2)(Bi + Bik - k_s S^2)}, \quad (A2) \]

\[ D_3 = \frac{(C_1)(Bi - 4S^2)}{(8S^2)(Bi + Bik - 4k_s S^2)}, \quad (A3) \]

\[ D_4 = \frac{C_1 + 2C_3}{4(k_r + 1)}, \quad (A4) \]

\[ D_5 = \frac{k_{se}(T_{w2} - T_{w1})}{H(q_{w1} + q_{w2})}, \quad (A5) \]

\[ D_1 = \frac{[C_1(Bi + Bik - k_s S^2)(Bikcosh^2(S) + Bicos^2(S) - 2k_s S^2)]}{(Bi)(Bi + Bik - k_s S^2)(Bi + Bik - 4k_s S^2)(k_r + 1)^2 \cosh\left(\frac{(|Bi/k_r|)(k_r + 1)}{2}\right)}, \quad (A1) \]

\[ D_6 = -\frac{[C_1(2Bikcosh^2(S) + 2Bik S^2 + 2Bicos^2(S) + 2Bis^2 - Bik + 4S^2 - Bi)] + (C_2)(Bicos(S))(k_r + 1)(Bi + Bik - 4k_s S^2) + (C_3)(4S^2)(Bik + Bi + 2)}{8Bis^2(k_r + 1)^2}, \quad (A6) \]

\[ E_1 = \frac{k_r [C_1(Bi + Bik - k_s S^2)(Bikcosh^2(S) + Bicos^2(S) - 2k_s S^2)] + (C_2)(Bicos(S))(k_r + 1)(Bi + Bik - 4k_s S^2) + (C_3)(Bi + Bik - k_s S^2)(Bi + Bik - 4k_s S^2)]}{(Bi)(Bi + Bik - k_s S^2)(Bi + Bik - 4k_s S^2)(k_r + 1)^2 \cosh\left(\frac{(|Bi/k_r|)(k_r + 1)}{2}\right)}, \quad (A7) \]
\[ E_2 = \frac{C_2 \text{Bi}}{(S^2)(B + \text{Bikr} - k_r S^2)^4} \]  
\[ E_3 = \frac{C_3 \text{Bi}}{(8S^2)(B + \text{Bikr} - 4k_r S^2)^3} \]  
\[ E_4 = \frac{C_1 + 2C_3}{4(k_r + 1)} \]  
\[ E_5 = \frac{\text{kw} (T_w - T_{w1})}{H(q_{w1} + q_{w2})} \]  
\[ E_6 = \left[ \left( C_1 \left( 2\text{Bikr} \cosh^2(S) + 2\text{Bikr} S^2 + 2\text{Bikr}^2 - \text{Bikr} S^2 - \text{Bikr} + \text{Bi} \right) \right) + \left( C_2 \left( 8\text{Bikr} \cosh(S) \right)(k_r + 1) + \left( C_3 \left( 4S^2 \text{Bikr} + \text{Bi} - 2k_r \right) \right) \right] \]  
\[ - \frac{8S^2(k_r + 1)^2}{(A12)} \]  

References


