Abstract—Dual-energy X-ray computed tomography (DECT) is a technique which is designed to allow the determination of energy-independent material properties. In this study, results of a computer simulation show that energy-dependent systematic errors exist in the values of attenuation coefficients synthesized using the basis material decomposition technique with acrylic and aluminum as the basis materials, especially when a high atomic number element such as iodine (e.g., from radiographic contrast media) is present in the body. The errors are reduced when an acrylic and an iodine-water mixture are used as the basis materials. We propose a simple theoretical model for the calculation of energy-dependent systematic errors using effective energies at the lower and higher energy windows of the X-ray spectrum used in the DECT system. The calculated errors agree well with the errors observed in the simulation. These results suggest that the observed systematic errors are predominantly due to the energy dependence of the basis material coefficients.

I. INTRODUCTION

Dual-energy X-ray computed tomography (DECT) [1], [2] is a technique designed to allow the determination of energy-independent material properties. This technique has been applied in quantitation of bone mineral content [3] and, more recently, in the acquisition of quantitatively accurate attenuation maps for attenuation correction in single-photon emission computed tomography (SPECT) [4]. In DECT, projection data are acquired using two X-ray energy spectra, and quantitatively accurate attenuation coefficient distribution can, in principle, be reconstructed at any given X-ray energy using the basis material decomposition (BMD) technique. The success of this technique depends on the accuracy of the assumption that the mass attenuation coefficient $\mu_\alpha(E)$ of any material $\alpha$ at any given X-ray energy $E$ can be expressed as a linear combination of the mass attenuation coefficients $\mu_1(E)$ and $\mu_2(E)$ of two basis materials 1 and 2

$$\mu_\alpha(E) = c_{\alpha 1}\mu_1(E) + c_{\alpha 2}\mu_2(E) \quad (1)$$

where the basis material coefficients $c_{\alpha 1}$ and $c_{\alpha 2}$ are assumed to be independent of energy $E$. However, the fitting of experimental values of the attenuation coefficients to (1) shows that the coefficients are, in general, energy-dependent [5], [6]. Thus, the synthesized values of mass attenuation coefficients using the BMD technique are expected to suffer from energy-dependent systematic errors.

The analysis of such energy-dependent systematic errors in dual-energy imaging has rarely been studied. Recently, Gingold and Hasegawa [6] had investigated such errors influencing the measurements in bone mineral densification using dual-energy X-ray absorptiometry. In this present study, we report on our investigation of the energy-dependent systematic errors in the reconstructed monoenergetic attenuation distribution map obtained from DECT with the BMD technique, particularly when a high atomic number element such as iodine (e.g., from radiographic contrast media) is present in the body. We have proposed a simple theoretical model using equivalent monoenergetic X-ray spectra to calculate the magnitude of the systematic errors due to the energy dependence of the basis material coefficients. Computer simulations were performed to study these errors and to verify the theoretical model. The simulations were carried out using a ray-tracing algorithm with a third-generation fan beam geometry. Two mathematical phantoms, one comprised of two concentric circular regions and another configured to resemble a thorax, were used [see Fig. 1(a) and (b)]. In both phantoms, the central circular region consisted of a mixture of iodine and water. Four different iodine concentrations were evaluated, i.e., 0%, 1%, 5%, and 10% by weight. Three sets of X-ray spectra were employed in the simulations, namely, dual monoenergetic spectra, a bimodal spectrum, and dual kVp spectra. Two sets of basis materials were compared. The first set was the conventional acrylic/aluminum basis set. In the second set, aluminum was replaced with an iodine-water mixture containing 20% iodine by weight (hereafter called $I_{20\%}$). In the reconstruction stage, the attenuation coefficients were synthesized within the medical diagnostic energy range (40–150 keV).

Our simulation results show that the magnitude of the systematic errors depends on the materials present in the body, the choice of the basis materials, the X-ray spectra used, and the energy at which the attenuation coefficients are synthesized. In particular, with the acrylic/aluminum basis set the systematic errors are large when a high atomic number element (e.g., iodine in X-ray contrast media which has a K-absorption edge at 33 keV) is present in the body. The errors are significantly reduced when the iodine-water mixture is chosen as one of the basis materials. Overall, the theoretical model for calculating the energy-dependent systematic errors agrees very well with the simulation results. These results suggest that the observed systematic errors are predominantly due to the energy dependence of the basis material coefficients.


Energy-Dependent Systematic Errors in Dual-Energy X-Ray CT

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II. ENERGY-DEPENDENT SYSTEMATIC ERRORS IN DECT

We propose a simple model for evaluating the energy-dependent systematic errors from the knowledge of the effective energies of the X-ray spectra used in DECT. For simplicity, we will consider the case when the phantom consists of only one single material. The analysis can easily be extended to include more than one material. The systematic errors in the synthesized attenuation coefficients arise from the energy dependence of the basis material coefficients. However, in any DECT system, a constant value for each basis material coefficient would be obtained. If \( c_{\alpha 1} \) and \( c_{\alpha 2} \) are the energy-independent estimates of the basis material coefficients \( c_{\alpha 1}(E) \) and \( c_{\alpha 2}(E) \), then the systematic errors in the attenuation coefficient synthesized at energy \( E \) is

\[
\Delta \mu_{\alpha}(E) = (c_{\alpha 1} - c_{\alpha 1}(E)) \mu_1(E) + (c_{\alpha 2} - c_{\alpha 2}(E)) \mu_2(E) \\
= (c_{\alpha 1} \mu_1(E) + c_{\alpha 2} \mu_2(E)) - \mu_{\alpha}(E). \tag{2}
\]

The values of \( c_{\alpha 1} \) and \( c_{\alpha 2} \) obtained would depend on the effective energies of the X-ray spectra in the two energy windows used in acquisition of the projection data. If the X-ray spectra are two monoenergetic peaks at \( E_L \) and \( E_H \), where \( L \) and \( H \) denote the low and high energy in the spectra, it can be shown using simple algebra that the values for \( c_{\alpha 1} \) and \( c_{\alpha 2} \) are

\[
c_{\alpha 1} = [\mu_2(E_H) \mu_1(E_L) - \mu_2(E_L) \mu_1(E_H)] / D \tag{3}
\]

\[
c_{\alpha 2} = [-\mu_1(E_H) \mu_1(E_L) + \mu_1(E_L) \mu_1(E_H)] / D \tag{4}
\]

where

\[
D = \mu_1(E_L) \mu_2(E_H) - \mu_2(E_L) \mu_1(E_H). \tag{5}
\]

We postulate that the values of \( c_{\alpha 1} \) and \( c_{\alpha 2} \) obtained from a DECT system using polyenergetic X-ray spectra can also be similarly calculated using (3) and (4) for the two energy windows. Note that (3) and (4) suggest that \( c_{\alpha 1} \) and \( c_{\alpha 2} \) are dependent upon the values of the attenuation coefficients of the basis materials and the unknown material at the effective energies of the spectra. Consequently, errors arising in the reconstruction of the attenuation coefficients in (1) would depend on the choice of \( E_L \) and \( E_H \) for the X-ray spectra used. We have chosen the mean photon energies of the entrance spectra [6] to be the effective energies \( E_L \) and \( E_H \) for the purpose of calculating the energy-dependent systematic errors. These effective energies are defined by

\[
E_x = \int_{E_{x1}}^{E_{x2}} E S_x(E) \, dE / \int_{E_{x1}}^{E_{x2}} S_x(E) \, dE. \tag{6}
\]

Here, the subscript \( x \) denotes \( L \) or \( H \), and \( S_x(E) \) is the X-ray spectrum of the corresponding energy window.

The computer simulation was carried out using a ray-tracing algorithm with a third-generation fan beam geometry CT system [8], [9]. Two sets of phantoms were evaluated. Fig. 1(a) shows a simple mathematical phantom (hereafter called the simple circular phantom) comprising two concentric circles. The outer circular region was water-filled. The inner circular region contained an iodine-water mixture. For the second phantom, we used a thorax-like phantom [4], [8] as shown in Fig. 1(b) and Table I. For both phantoms, four different concentrations of iodine were evaluated, namely, 0, 1, 5, and 10% by weight. The lower concentrations (≤5%) were typical values found in the body after the administration of iodinated contrast media. The higher values were used to study the accuracy and to determine the limit of validity of the proposed theoretical model and the BMD technique.

In the simulation, three different sets of energy spectra were employed for the simple circular phantom. The first was a set of dual-monoenergetic spectra peaked at 47 and 99 keV. The second was a typical bimodal spectrum used in dual-energy X-ray measurements [6], obtained from a 120-kVp X-ray spectrum with additional filtration using a 1-mm thick gadolinium filter (K-absorption edge at about 50 keV). The lower energy window of the bimodal spectrum extended from 14 to 50 keV with an effective energy of \( E_L = 47 \) keV. The upper energy window had an effective energy of \( E_H = 99 \) keV and extended from 51 to 120 keV. The third was a
set of dual-kVp spectra, also commonly used in dual-energy measurements [11]. The low energy 80-kVp X-ray spectrum was filtered with 1-mm copper, while the high-energy 140-kVp X-ray spectrum was filtered with 3-mm tin. The added filtration minimized overlapping and provided a reasonable energy separation between the two spectra. The lower 80-kVp spectrum spanned from 26 to 80 keV and the higher 140-kVp spectrum from 63 to 140 keV. Both spectra overlapped slightly from 63 to 80 keV. These broad-band spectra were obtained from tabulated data of Birch et al. [7]. For the thorax-like phantom, only the bimodal spectrum was used in the simulation.

For both phantoms, two sets of basis materials were evaluated. The first set was the conventional set which used acrylic to mimic soft tissue components and aluminum to mimic the bones. However, if iodine is present in the body at a substantial amount, these two materials are not sufficient to represent the energy variation of the iodine absorption coefficient since iodine has a K-absorption edge at 33 keV. Large error would then be introduced. In the second set of our basis materials, an iodine-water mixture containing 20% iodine by weight replaced aluminum as a basis material. We found that this basis set reduced the energy-dependent systematic error when iodine was present.

Details of the dual-energy simulation have been described elsewhere [9]. In the simulation, the exact attenuation coefficients of the materials used were calculated using an algorithm which fitted fourth-order polynomials to the logarithms of tabulated values of mass attenuation coefficients [10]. In the reconstruction stage, the line integral of the basis material coefficients were evaluated from the projection data using a fifth-order polynomial whose coefficients were obtained from a calibration scan of known thicknesses of the basis materials [2], [6]. Images of the basis material coefficients were then generated from the line integrals of the basis coefficients using the convolution backprojection algorithm. Monoenergetic attenuation maps were synthesized using (1) for energy ranging from 40 to 150 keV. The lower limit was chosen to exclude the iodine K-edge. The mean value of the synthesized attenuation coefficient for each region in the phantom was determined.

III. RESULTS

In the computer simulations of DECT, the percentage systematic error as a function of energy $E$ was determined by taking the difference between the synthesized value $\mu_{CS}(E)$ and the actual value $\mu_{actual}(E)$ of the attenuation coefficient, normalized by the latter value

$$\text{Percent error} = 100 \times \left( \frac{\mu_{actual}(E) - \mu_{CS}(E)}{\mu_{actual}(E)} \right)$$

(7)

The percentage error predicted by the theoretical model was similarly calculated. The goodness-of-fit for the attenuation coefficients ($\mu_{TM}$) was quantified by the root-mean-square parameter (rms)

$$\text{rms} = \left[ \frac{1}{N} \sum_{E} \left( \frac{\mu_{TM}(E) - \mu_{CS}(E)}{\mu_{CS}(E)} \right)^2 \right]^{1/2}$$

(8)

where $\mu_{TM}$ was calculated from (1), (3), and (4) using the effective energies determined from (6), and $\mu_{CS}$ was determined from computer simulation.

Figs. 3–5 show computer simulation results of energy-dependent systematic errors for the synthesized attenuation coefficients in the iodinated inner circular region of the simple
circular phantom. These results were obtained using the three different sets of spectra and the two different sets of basis materials. In Fig. 6, the energy-dependent errors are shown for the attenuation coefficients in the outer circular region using the bimodal spectrum when iodine is present in the inner circular region at various concentrations.

Figs. 7–10 show the plots of the percent errors of the attenuation coefficient calculated using the theoretical model versus computer simulation for the inner region (0% and 10% iodine) of the simple circular phantom, using the bimodal spectrum with (a) acrylic/aluminum and (b) acrylic/20% basis sets. The dashed line is the line-of-identity.

Table II presents typical results of the goodness-of-fit between the theoretical model and the computer simulation quantified by the rms parameters defined in (8).

Fig. 11–14 show computer simulation results of the energy-dependent systematic errors for the synthesized attenuation coefficients in the “heart,” “main-body,” “sternum,” and “spine” regions of the thorax-like phantom. The percent errors of the attenuation coefficient calculated using the theoretical model agree closely with the values obtained from computer simulation for the thorax-like phantom.

IV. DISCUSSION AND CONCLUSION

Our simulation results show that energy-dependent systematic errors exist, the magnitude of which depend on the materials present in the body, the choice of the basis materials, the X-ray spectra used, and the energy at which the attenuation coefficients are synthesized. The systematic errors of the synthesized attenuation coefficient values at the iodinated
region are large when acrylic and aluminum are used as the basis materials. The magnitude of the error increases with increasing iodine concentration. However, it is interesting to note that at all iodine concentrations evaluated, the errors are close to zero (Fig. 3) at energy near to the effective energies $E_L$ and $E_H$ of the spectra used. It can be shown using (1)–(5) that the errors are zero at the two effective energies if dual monoenergetic spectra are used. However, it is not obvious theoretically that this should also be true for polyenergetic spectra. The observation that the errors almost vanish at energy close to the effective energies for the polyenergetic spectra (Figs. 4, 5, 11, and 13) suggests that (3) and (4) for dual monoenergetic spectra are used. However, it is not obvious theoretically that this should also be true for polyenergetic spectra.

The variations of the systematic errors with energy for the three different spectra evaluated exhibit a similar trend. In particular, Fig. 3 for the dual monoenergetic spectra is practically the same as the corresponding Fig. 4 for the bimodal spectrum with the same effective energies. This observation supports our theoretical model for predicting the energy-dependent systematic errors using the two effective energies $E_L$ and $E_H$ of the spectra used in DECT. Fig. 3 shows that the energy-dependent systematic errors exist even when dual monoenergetic spectra are used in DECT. The errors
cannot be eliminated because of the violation of the intrinsic assumption of the dual BMD technique that only two functions are sufficient to synthesize the energy dependence of the attenuation coefficient of any material. However, the errors can be reduced by using the appropriate choice of basis materials to match the materials present in the body.

The calculated energy-dependent systematic errors using the theoretical model generally agree very well with the simulation results. These results suggest that the observed systematic errors in DECT are predominantly due to the energy dependence of the basis material coefficients.

We have chosen the mean entrance photon energies at the lower and higher energy windows as the effective energies for the polyenergetic spectra in the theoretical model for calculating the systematic errors. This choice is not based on any theoretical principle but is used out of convenience and simplicity. In fact, other choices of the effective energies, such as the modal energies and the mean exit photon energies, may be used. Our choice seems to be satisfactory for lower iodine concentrations (5% or less). At higher iodine concentrations, the agreement between the theoretical model and simulation results is not as good.

In conclusion, we have demonstrated that energy-dependent systematic errors exist in DECT even when dual monoenergetic spectra are used. This error is due to the violation of the two-dimensionality assumption of the BMD model. The errors are especially severe when a high atomic number element such as iodine in radiographic contrast media is present in the body. These errors can be reduced significantly with proper selection of basis materials.

The study presented in this paper is theoretical in nature and is limited to simple phantoms. A more anatomically realistic phantom such as the three-dimensional MCAT phantom [12] can be employed for further investigation. Verification of the theoretical results by experiment will also be the subject of future study.

REFERENCES


