Correction of Energy-Dependent Systematic Errors in Dual-Energy X-ray CT using a Basis Material Coefficients Transformation Method

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Abstract

Computer simulation results from our previous studies showed that energy dependent systematic errors exist in the values of attenuation coefficient synthesized using the Basis material decomposition technique with acrylic and aluminum as the basis materials, especially when a high atomic number element (e.g., iodine from radiographic contrast media) was present in the body. The errors were reduced when a basis set was chosen from materials mimicking those found in the phantom. In the present study, we employed a basis material coefficients transformation method to correct for the energy-dependent systematic errors. In this method, the basis material coefficients were first reconstructed using the conventional basis materials (acrylic and aluminum) as the calibration basis set. The coefficients were then numerically transformed to those for a more desirable set of basis materials. The transformation was done at the effective energies of the low and high energy windows of the x-ray spectrum. With this correction method using acrylic and an iodine-water mixture as our desired basis set, computer simulation results showed that accuracy of better than 2% could be achieved even when iodine was present in the body at a concentration as high as 10% by mass. Simulation work had also been carried out on a more inhomogeneous 2D thorax phantom of the 3D MCAT phantom. The results of the accuracy of quantitation were presented here.

I. INTRODUCTION

Dual-energy x-ray Computed Tomography (DECT) is a technique which allows the determination of energy-independent material properties and to acquire quantitatively accurate monoenergetic attenuation distribution images free from beam hardening artifacts [1, 2]. This technique has applications in quantitation of bone mineral content [3] and in the acquisition of quantitatively accurate attenuation maps for attenuation correction in SPECT [4]. For the purpose of attenuation correction, separate transmission x-ray scans are carried out using two x-ray energy spectra. The quantitatively accurate attenuation coefficient distribution can then be reconstructed at any given x-ray energy using the basis material decomposition (BMD) technique.

The BMD technique is based on the premise that the linear attenuation coefficient $\mu_\xi(E)$ of any material $\xi$ at any given x-ray energy $E$ can be expressed as a linear combination of the linear attenuation coefficients $\mu_1(E)$ and $\mu_2(E)$ of two basis materials 1 and 2,

$$\mu_\xi(E) = c_{\xi1}\mu_1(E) + c_{\xi2}\mu_2(E),$$

where the basis material coefficients $c_{\xi1}$ and $c_{\xi2}$ are assumed to be independent of energy $E$. However, these coefficients are, in general, energy dependent [5, 6]. Hence, the synthesized values of linear attenuation coefficients using the BMD technique are expected to suffer from energy dependent systematic errors.

In an earlier work [7], we investigated the systematic errors in the attenuation coefficients synthesized by the BMD technique via computer simulation of the DECT system. Our results showed that the errors were significant, particularly when a high atomic number element (such as iodine in x-ray contrast media) was present in the body. When quantifying attenuation coefficient values in the iodinated contrast regions, the systematic errors were found to vary with the synthesized (display) energies as well as the concentration of the iodine present in the region. At 10% iodine concentration, a maximum of about 15% systematic error occurred at the synthesized energy of 140keV. This is greater than the statistical noise level of 4% to 6% (precision error) noted in various dual-energy procedures [11]. The systematic errors were significantly reduced when basis materials mimicking the materials present in the phantom, e.g., acrylic and an iodine-water mixture, replaced the conventional acrylic/aluminum calibration materials. In practice, the acrylic/aluminum basis set offers several practical advantages such as low cost, ease of handling and accuracy of machining to required dimensions for accurate calibration. On the other hand, basis materials that mimic the body materials radiographically may not have these desirable properties.

In this paper, we describe our attempt to correct for the energy-dependent systematic errors using a basis material coefficients transformation method. Previous authors [2, 8] have hinted on the idea of basis coefficients transformation, but to date, no systematic study of this subject has been published in the literature. The basis coefficients transformation method essentially transforms the reconstructed set of the conventional acrylic/aluminum basis coefficients to one of a more desirable set whose materials properties mimic closely the materials present in the slice to be quantified. Two mathematical phantoms were used in our computer simulation study: a simple thorax-like phantom and
a more realistic 2D slice of the thorax region from the 3D MCAT phantom [9]. Iodine at various concentrations was present in the heart region of the phantom. The distribution of the basis coefficients was first reconstructed using the conventional acrylic/aluminum as the basis materials. The basis materials transformation was applied to transform the coefficients to correspond to a more desirable basis set. We have evaluated this method using both a dual-monoenergetic spectrum and a bimodal spectrum.

The basis materials transformation method succeeded in reducing the energy-dependent systematic errors of the reconstructed attenuation coefficient. For instance, using acrylic and an iodine-water mixture as the desired basis set, our computer simulation results showed that accuracy of 2% or better could be achieved in the quantification of the iodinated region even when iodine concentration as high as 10% was present. In quantitative imaging, achieving accuracy better than 2% can improve image quality and diagnostic ability. This is particularly so in quantitative Single Photon Emission CT (SPECT) imaging because any significant improvement in diagnosis demands that compensation methods, such as the Maximum Likelihood Expectation Maximization (MLEM), require a good knowledge and accuracy of the attenuation coefficient distribution through experimental measurement [12].

II. MATHEMATICAL FORMULATION

We first consider a DECT system using an x-ray spectrum consisting of two monoenergetic peaks at $E_L$ and $E_H$. The linear attenuation coefficient $\mu_\xi(E)$ for a material $\xi$ present in the body at these two energies can then be expressed as

$$\mu_\xi(E_L) = c_{\xi 1}\mu_1(E_L) + c_{\xi 2}\mu_2(E_L),$$

(2)

$$\mu_\xi(E_H) = c_{\xi 1}\mu_1(E_H) + c_{\xi 2}\mu_2(E_H),$$

(3)

where $\mu_1(E)$ and $\mu_2(E)$ are the energy dependent attenuation coefficients of the two basis materials 1 and 2, $c_{\xi 1}$ and $c_{\xi 2}$ are the basis coefficients of the material $\xi$ for this basis set. If a second set of basis materials were used, then the linear attenuation coefficient of the material $\xi$ at the same energies can be expressed in terms of the new basis coefficients $c'_{\xi 1}$ and $c'_{\xi 2}$ as

$$\mu_\xi(E_L) = c'_{\xi 1}\mu'_1(E_L) + c'_{\xi 2}\mu'_2(E_L),$$

(4)

$$\mu_\xi(E_H) = c'_{\xi 1}\mu'_1(E_H) + c'_{\xi 2}\mu'_2(E_H),$$

(5)

where $\mu'_1(E)$ and $\mu'_2(E)$ are the linear attenuation coefficients of the two basis materials constituting the new basis set. The two sets of basis coefficients ($c_{\xi 1}$, $c_{\xi 2}$) and ($c'_{\xi 1}$, $c'_{\xi 2}$) are related by the transformation

$$\begin{pmatrix} c'_{\xi 1} \\ c'_{\xi 2} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} c_{\xi 1} \\ c_{\xi 2} \end{pmatrix},$$

(6)

where

$$a_{11} = [\mu'_1(E_H)\mu'_2(E_L) - \mu'_2(E_L)\mu'_1(E_H)] / D,$$

(7)

$$a_{12} = [\mu'_1(E_L)\mu'_2(E_H) - \mu'_2(E_H)\mu'_1(E_L)] / D,$$

(8)

$$a_{21} = [\mu'_1(E_H)\mu'_2(E_L) - \mu'_2(E_L)\mu'_1(E_H)] / D,$$

(9)

$$a_{22} = [\mu'_1(E_H)\mu'_1(E_L) - \mu'_2(E_L)\mu'_1(E_H)] / D,$$

(10)

and

$$D = \mu'_1(E_L)\mu'_2(E_H) - \mu'_2(E_H)\mu'_1(E_L).$$

(11)

The transformation in Eq. (6) have been derived assuming that the basis coefficients are energy independent. This assumption is not strictly valid, especially when a high atomic number element is present in the body. Our previous studies [7] have shown that the systematic errors due to the energy dependence of the basis coefficients can be significant. However, if an appropriate set of basis materials were chosen, then the errors can be reduced. Our results presented in the following sections show that the systematic errors can also be reduced without physically using the more desirable basis materials in the calibration scan. The calibration scan can be performed using the conventional acrylic/aluminum basis materials. The basis coefficients for the acrylic/aluminum basis set can be numerically transformed using Eq. (6) to those corresponding to the more desirable basis set.

For the polyenergetic bimodal spectrum typically used in DECT systems, we postulate that the transformation in Eq. (6) can still be applied, but with $E_L$ and $E_H$ replaced by some suitably defined effective energies in the lower and upper energy windows respectively. We have used the mean entrance energy as the effective energy for each energy window.

III. COMPUTER SIMULATION

A schematic diagram of the computer simulation is illustrated in Fig. 1. The simulation of the DECT system was carried out using the ray-tracing algorithm with a third generation cone beam geometry. Details of the simulation and reconstruction of the basis set coefficients from the thickness images of the calibration materials have been described elsewhere [7]. Conventionally, the attenuation coefficients are synthesized directly from the reconstructed basis material coefficients. In the basis material coefficients transformation method, an additional step is included. This step numerically transforms the basis material coefficients for the conventional basis set actually used in the calibration scan to another set corresponding to the basis materials having similar radiographic properties as the materials found in the phantom.

Two types of x-ray spectra were employed in the simulation. The first one was a hypothetical dual monoenergetic spectrum peaked at 47 keV and 99 keV and the second was a bimodal spectrum obtained from a 120 kVp spectrum filtered with 1-mm gadolinium. The effective energies of the high and low energy windows of the bimodal spectrum were evaluated using the equation,

$$E_x = \left[ \frac{\int_{E_x}^{\infty} S_x(E) dE}{\int_{E_x}^{\infty} S_x(E) dE} \right]$$

(12)

Here, the subscript $x$ denotes $L$ or $H$ for the low or high energy window, $S_x(E)$ is the x-ray spectrum of the corresponding energy window. The effective energies were
found to be $E_L = 47$ keV and $E_H = 99$ keV, for the bimodal spectrum.

The thorax-like phantom and a slice through the thorax of the MCAT phantom are shown in Fig. 3 and Fig. 4. The materials present in the phantoms are listed in Table 1. Several concentrations (at 0%, 1%, 5% and 10% of iodine by mass) of the iodine-water mixture present in the heart region of both phantoms were tested in the simulation. While concentrations of 1% and below are normally found in the body after the administration of iodinated contrast media, values at 1% and above were included in our simulation studies for the following reasons. Systematic errors in the small iodine concentration range were usually not significant \[10\]. However, these errors are expected to be much higher at high iodine concentration. Hence, it is worthwhile to investigate the effectiveness of the transformation method in reducing the systematic errors introduced at high concentration range. Furthermore, we would like to generate results that would be applicable to a wide range of systems.

![Fig. 1: Flow-chart of the computer simulation.](image1)

![Fig. 2: The two spectra (a) hypothetical dual monoenergetic and (b) bimodal, used in the simulation.](image2)

![Fig. 3: The ‘thorax-like’ phantom](image3)

![Fig. 4: The MCAT phantom showing the thorax region. P and Q are two end points on the dotted line.](image4)

<table>
<thead>
<tr>
<th>Region</th>
<th>Material</th>
<th>$\rho$ (g/cm$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heart chamber</td>
<td>Iodine-water mixture, with varying iodine conc. of 0%, 1%, 5% and 10% by mass.</td>
<td>1.00 (0%), 1.01 (1%), 1.04 (5%), 1.08 (10%)</td>
</tr>
<tr>
<td>Lung</td>
<td>void material, $\mu = 0$</td>
<td>zero</td>
</tr>
<tr>
<td>Spine</td>
<td>Bone-mineral</td>
<td>3.06</td>
</tr>
<tr>
<td>Sternum &amp; ribs</td>
<td>Compact bone</td>
<td>1.85</td>
</tr>
<tr>
<td>Main body</td>
<td>Water</td>
<td>1.00</td>
</tr>
<tr>
<td>Heart Wall</td>
<td>Water</td>
<td>1.00</td>
</tr>
</tbody>
</table>

In all cases, the conventional acrylic/aluminum basis set was used in the calibration scan, and the basis coefficients corresponding to acrylic/aluminum were first generated. Then Eq. (6) was applied to numerically transform the basis coefficients to those corresponding to a more desirable set of basis materials. Four different sets of the more desirable basis materials were studied: (1) acrylic and an iodine-water mixture comprising 20% iodine by weight (I$_{20\%}$), (2) water and a pure iodine solution, (3) water and bone.
mineral [6], and (4) water and calcium [8]. Errors in the synthesized basis coefficients values using the transformed basis coefficients are quantified in the usual way,

$$\% \text{error} = \frac{\mu_{\text{actual}}(E) - \mu_{\text{simulated}}(E)}{\mu_{\text{simulated}}(E)} \times 100$$  \hspace{1cm} (13)

where \( \mu_{\text{actual}} \) is the value of the exact attenuation coefficient value.

IV. RESULTS AND DISCUSSIONS

Fig. 5 shows typical graphs of the percent error versus energy when a bimodal spectrum was used for the simple thorax phantom. The attenuation coefficients were synthesized using the basis coefficients corresponding to the acrylic/aluminum basis set. The results after transformation to the acrylic/I\textsubscript{20\%} basis set are shown in Fig. 6. The maximum error was about 15\% in the iodinated region for the attenuation coefficient synthesized using the acrylic/aluminum basis set. After basis material transformation, the maximum error was significantly reduced to about 1.7\%. Incidentally, lower errors were obtained after transformation when a dual monoenergetic spectrum (not shown) was used. The maximum error in this case was about 1.2\%. The errors after transformation in the “spine” and “sternum” regions were generally higher than those in the other two regions. This is because the attenuating property of I\textsubscript{20\%} is very different from the materials in the “spine” and “sternum” regions, so large errors (peaked at around 4\% and 2.5\% respectively) are expected to occur.

With acrylic replaced by water and I\textsubscript{20\%} by pure iodine as the desired basis materials, there was no apparent change of accuracy in the iodinated region (results not shown). This suggests that the differences between the attenuating properties of iodine and I\textsubscript{20\%} are not very significant.

Fig. 6: Percent error of the simulated attenuation coefficient values (after transformation) versus x-ray energy obtained from the thorax-like phantom using the bimodal spectrum for (a) “heart” containing iodine at 0\%(\(\bigcirc\)), 1\%(\(\bullet\)), 5\%(\(\bigodot\)) and 10\%(\(\bullet\)); (b) main body (water); (c) “sternum” (compact bone) and (d) “spine” (bone mineral). The transformed basis materials were acrylic and aluminum.

Fig. 7: Percent error of the simulated attenuation coefficient values (after transformation) versus x-ray energy obtained from the thorax-like phantom using the bimodal spectrum for (a) “heart” containing iodine of 0\%(\(\bigcirc\)), 1\%(\(\bullet\)), 5\%(\(\bigodot\)) and 10\%(\(\bullet\)); (b) main body (water); (c) “sternum” and (d) “spine”. The transformed basis materials were water and bone mineral.

Fig. 7 illustrates an example of an attempt made to improve the accuracy in the other parts of the phantom outside the iodinated region. By using water to mimic soft-
tissue and bone mineral or calcium (results not shown) replacing iodine as the second basis material, significant improvement in the accuracy of bone mineral or compact bone quantification was observed. Comparing Fig. 6d and Fig. 7d for the bone mineral region, the percent error was reduced from a maximum of about 4% to 0.4%. Similarly, the compact bone region shows a reduction from a maximum of about 2% to 0.5%. However, the accuracy in the iiodinated region was compromised. The errors peaked at about 10% in this region.

For the thorax-like phantom using the dual-monoenergetic spectrum, the errors were practically zero around the peak energies of the spectrum (results not shown). However, the conclusion does not hold true if a bimodal spectrum is used instead, as shown in Fig. 6a. In principle, since the basis material coefficients transformation operates at the effective energies of the spectra, zero error should occur at these energies for any material present in the phantoms. However, it was observed that zero error occurred at around the effective energies of the bimodal spectrum only when the selected basis materials exhibit an absence of any close resemblance to the attenuating property of the material present in the phantom.

In quantitative SPECT imaging, improved diagnosis demands that compensation methods such as MLEM requires a good knowledge of the attenuation coefficient distribution through experimental measurement. If accurate attenuation map can be obtained, this would result in more improved quantitative accuracy in the reconstructed image and consequently improved diagnostic capability [12]. As an illustration of the applicability and effectiveness of the transformation method for such attenuation compensation in SPECT imaging [4], Fig. 8 shows our results of profile plots of the reconstructed attenuation map at 140keV (corresponding to the gamma-ray energy emitted by radionuclide 99mTc) obtained from the MCAT phantom. These results were obtained using the bimodal spectrum with acrylic/aluminum as the calibration materials, and then transformed to the acrylic/I30% basis set. The plots showed that the basis materials transformation method reduced the energy dependent systematic error in the iodinated and bone regions significantly at iodine concentration ranging from 5 to 10%. The deviations exhibited were fairly consistent with those obtained using the simple thorax-like phantom. At iodine concentration of 10% in the heart region, the error in the attenuation coefficients synthesized (at 140 keV) using the conventional acrylic/aluminum basis set was about 19%. In comparison, the error was reduced to about 4% when the basis coefficients were transformed to the acrylic/I30% basis set. Other recently reported methods such as the uniform scaling of attenuation coefficients method [9] had shown to give accurate estimates of the soft tissue attenuation map and the resultant estimates of the myocardial radioactivity concentration was reported accurate to within 9%. However, it could only achieve an overall accuracy of 21% to 42% for the spine region and 58% for the ribs. While the degree of accuracy in the presence of high iodine concentration in soft tissue regions was not demonstrated with this method, we believe that the uniform scaling method is expected to perform much poorer than the basis material coefficients transformation method.

![Fig. 8: Attenuation coefficient profiles of the MCAT phantom (along the line PQ in Fig. 4) at 140keV using the bimodal spectrum. Iodine concentration in the heart chamber was (a) 0%, (b) 1%, (c) 5% and (d) 10%. The exact profile (–), the profiles synthesized from the conventional acrylic/aluminum basis set (■) and the transformed acrylic/I30% basis set (♦) are shown in each case.](image-url)

Overall, our simulation results suggest that the basis material coefficient transformation method can reduce the systematic errors. This is not surprising because the proposed method is based on two fundamental assumptions. First, that a solution to the basis material coefficients can be analytically
determined only if these coefficients are dependent on material properties and not energy dependent. Second, that there is a set of basis materials which can best describe the attenuation coefficients of the different materials in the phantom. Note also that the matrix transformation from one basis material set to another is basically a linear operation which does not violate the fundamental principle of Dual-energy measurements. Thus, the success of this method would rest on the determination of the effective energies from the choice of the X-ray spectra used, the basis materials and the materials present in the slice. Indeed, significant reduction is possible by judicious selection of the desired basis set for the final synthesis of the attenuation coefficients of the materials in the slice on the condition that at least one component in the basis set mimicks or closely resembles the material properties of the material to be imaged.

A question arises concerning the use of the mean entrant energies as the effective energies $E_L$ and $E_H$ of the two energy windows in the bimodal spectrum. Such effective energies may be more appropriately defined to be the mean exit energies since the exit x-ray photons get detected at the detectors. In our simulation, we did not use this because the computation is more complex. In fact, the choice of using the entrant energies of the polyenergetic spectrum is not based on any theoretical principle, but is used out of convenience and simplicity. Other choice of the effective energies, such as the modal energies may be considered.

V. REFERENCES


