Critical length of collagen fibrils in extracellular matrix

Extracellular matrix (ECM) can be considered as a biological example of a fibre-reinforced composite material in which collagen fibrils provide tensile reinforcement (Hukins and Aspden, 1985). The fibrils are surrounded by a weak proteoglycan (PG) gel and may have other molecules, such as decorin and biglycan, closely associated with them (Hedbom and Heinegard, 1993; Roughley and Lee, 1994). Since the collagen fibril is much larger than these macromolecules, they may be considered as a continuum, which interacts with the fibril surface. The concept of "critical length", as used in the theory of fibre reinforcement (Cottrell, 1964; Kelly and Macmillan, 1986; Agarwal and Broutman, 1990), has then been applied to ECM in general (Hukins and Aspden, 1985) and to the ECM of specific tissues: annulus fibrosus of the intervertebral disc (Hickey and Hukins, 1982; Adams and Green, 1993), articular cartilage (Hukins et al., 1984) and uterine cervix (Aspden, 1988). The critical length, \( L_c \), is defined as the minimum length that a fibre must have for the stress at its centre to reach its fracture stress; if the length, \( L \), of a fibre is greater than \( L_c \), it is often supposed that there is a region in the centre of the fibre along which its axial tensile stress is constant and equal to the fracture stress (Cottrell, 1964; Hukins and Aspden, 1985; Kelly and Macmillan, 1986; Agarwal and Broutman, 1990). Here, we show that this supposition is not supported by a careful examination of theory but investigate how a plateau might appear in the stress distribution.

To develop the arguments, we consider tension to be applied to a tissue in the direction of an assembly of parallel fibrils. Such regions occur in many tissues such as tendon and articular cartilage; application of tensile stress also tends to orient fibrils (Aspden, 1986). Applied tension induces a shear stress, \( \tau \), at the interface between the PG gel and the fibril. This stress induces an axial tensile stress in the fibril (Kelly and Macmillan, 1986) that balances the applied tension; the higher the applied tension, the greater the magnitude of \( \tau \) and, hence, the stress within a fibril. We define the z-axis by the direction of the fibrils and consider a single fibril, of length \( L \), whose centre defines \( z = 0 \). Defining the origin at the centre of the fibril exploits the symmetry of the system but, as a result, some of the equations given here have a slightly different form to those given in the original papers. We use \( z \) to describe the distribution of \( \tau \) and \( \sigma \), along the length of a fibril, so that \(-L/2 \leq z \leq L/2\).

We first consider the case when the PG gel deforms plastically. Plastic deformation is important for two reasons. Firstly, it is the condition which leads to the concept of critical length. Secondly, it is likely to occur when the stress in the fibril \( \sigma \approx \sigma_f \), where \( \sigma_f \) is the fracture stress of collagen, since PG gels flow at a stress of about 100 kPa and \( \sigma_f \approx 100 \text{ MPa} \) (Hukins et al., 1984). For a cylindrical fibril, \( \sigma(z) \) rises linearly from the ends, where it is zero, to a maximum at the centre. This result can be clearly seen from the relationship

\[
\sigma(z) = \frac{\tau L}{r} \left[ 1 - \frac{2|z|}{L} \right], \tag{1}
\]

where \( r \) is the fibril radius (Kelly and Davies, 1965; Kelly and Tyson, 1965). Note that the form of Eq. (1) differs from that in the references cited because we define \( z = 0 \) to be at the fibril centre. Eq. (1) shows that \( \sigma(z) \) reaches a maximum value of

\[
\sigma(0) = \frac{\tau L}{r} \tag{2}
\]

at the centre of the fibril. Note that, in some publications (Aspden, 1994; Goh et al., 1999), Eq. (2) differs from that given here by a factor of 2, because a value of \( 2L \) has been assigned to the fibril length; Eq. (1) will then also differ for the same reason. The same result can be obtained from an analysis based on Filon’s solution for axial stress in an axisymmetric system (Aspden, 1994). For a cylindrical fibril, an expression may be found for the critical length from Eq. (2)

\[
L_c = r\sigma_f / \tau, \tag{3}
\]

where \( \sigma_f \) is considered to be the maximum stress which can be induced in the fibril. Then Eq. (1) is considered to apply only when \( L < L_c \); thus it is supposed that \( \sigma(z) \) reaches a plateau if \( L > L_c \) (Agarwal and Broutman, 1990; Kelly and Macmillan, 1986) so that \( \sigma(z) = \sigma_f \) when \( |z| \leq (L - L_c)/2 \) and \( L \geq L_c \). There is no theoretical justification for these suppositions. For a given applied
tensile stress, $\tau$ is a constant and acts over the whole length of the fibril. According to Eq. (2), increasing $L$ then increases $\sigma(0)$; $\sigma(0)$ will continue to increase until the fibril fractures. Thus the only justifiable interpretation of Eq. (3) is that $\sigma_f$ represents the fracture stress of the material of the fibre; so the fibre fractures when $L = L_c$. Thus, for effective reinforcement, $L$ should be large but less than $L_c$.

There are two reasons why there may be a plateau in $\sigma(z)$: the fibril is not cylindrical or the PG gel deforms elastically. Collagen fibrils in the ECM of at least some tissues, are tapered (Purslow and Trotter, 1992; Purslow and Trotter, 1994; Trotter et al., 1994; De Vente et al., 1997). In the case of a fibril with a straight taper (i.e. if it can be considered as two cones joined at their base) reinforcing a plastic gel, $\sigma(z)$ has a constant value of $\tau L/2r$ throughout the length of the fibre, where $r$ is now the radius when $z = 0$ (Goh et al., 1999, 2000). Thus there is now a plateau region for which $\sigma(z)$ is constant but it extends the length of the fibre. For the parabolic taper identified in vitro (Holmes et al., 1992), $\sigma(z)$ lies between the extremes of a linear increase towards the centre (cylindrical fibril) and a constant (straight taper) (Goh et al., 1999, 2000). For a cylindrical fibril reinforcing an elastic PG gel, $\sigma(z)$ increases from zero at the ends of the fibre to a maximum value at its centre; however, the increase is not linear. Then $\sigma(z)$ may be represented by a function of the form

$$\sigma(z) = C \left[ 1 - \frac{\cosh(\beta z)}{\cosh(\beta L/2)} \right], \quad (4)$$

where $C$ and $\beta$ are constants (Cox, 1952; Rosen, 1965; Nairn, 1997); note that choosing $z = 0$ to be at the centre of the fibril means that the form of Eq. (4) is slightly different to that in the papers cited. Eq. (4) shows that $\sigma(z)$ reaches a maximum value at $z = 0$. As $L \to \infty$, in Eq. (4), $\sigma(0) \to C$, i.e. $C$ can be identified with the maximum stress in an infinite fibril. When $C < \sigma_f$, an apparent plateau region can occur, depending on the relative values of $\beta$ and $L$.

In conclusion, when the length of a fibril reaches the critical length, it will fracture. If the PG gel deforms plastically, as expected, there will only be a plateau in the axial stress distribution if the fibril has a straight taper. In the event that the gel deforms elastically, it may be possible to observe a plateau in the axial stress distribution; however, the critical length, as conventionally defined by Eq. (3), does not then apply.

We thank Dr. Katharine Mathias for collaboration in early stages of this project, the University of Aberdeen for the award of a postgraduate studentship to KLG and the Medical Research Council (UK) for the award of a Senior Fellowship to RMA.


K.L. Goh
Department of Bio-Medical Physics and Bio-Engineering, University of Aberdeen, Foresterhill, Aberdeen, Scotland AB25 2ZD, UK

D.W.L. Hukins
Department of Bio-Medical Physics and Bio-Engineering, University of Aberdeen, Foresterhill, Aberdeen, Scotland AB25 2ZD, UK
E-mail address: d.hukins@biomed.abdn.ac.uk

R.M. Aspden
Department of Orthopaedic Surgery, University of Aberdeen, Foresterhill, Aberdeen, Scotland AB25 2ZD, UK

1Present address: Department of Biological Sciences, University of Stirling, Stirling, Stirling FK9 4LA, UK.